Eur. Phys. J. A **15**, 449–454 (2002)
DOI 10.1140/epja/i2002-10064-2

THE EUROPEAN PHYSICAL JOURNAL A

Configuration-mixed effective SU(3) symmetries

P.O. Hess[1,a], A. Algora[2,3], M. Hunyadi[2,4], and J. Cseh[2]

[1] Instituto Ciencias Nucleares, UNAM, A.P. 70-543, 04510, México, D. F., Mexico
[2] Institute of Nuclear Research of the Hungarian Academy of Sciences, H-4001 Debrecen, Pf. 51, Hungary
[3] IFIC-Universitat Valencia, Apartado Oficial 22085, 46071 Valencia, Spain
[4] KVI, 9747 AA Groningen, The Netherlands

Received: 23 April 2002/ Revised version: 10 July 2002 /
Published online: 10 December 2002 –
Communicated by A. Molinari

Abstract. The procedure of Jarrio *et al.* (Nucl. Phys. A **528**, 409 (1991)) for the determination of the effective $SU(3)$ symmetry of nuclear states is extended to small deformations and to oblate nuclei. Self-consistency checks are carried out both for light and for heavy nuclei.

PACS. 21.60.-n Nuclear-structure models and methods – 21.60.Fw Models based on group theory

1 Introduction

The effective symmetry [1] is one of the most general ones in quantum mechanics. It is a symmetry of the eigenvalue equation, when neither the (Hamiltonian) operator is symmetric (scalar), nor its eigenvectors (transform according to some irreducible transformations). Yet it may act, and have important physical consequences.

The mathematical reason for this surprising situation is provided by the embedded representation [2]. An embedded representation is obtained by calculating the matrix elements of the operators between vectors which are special linear combinations of those belonging to irreducible representations (irrep). The linear combinations are special in the sense that their coefficients are the same for several vectors. When the summation and the internal product operations are exchangeable (either exactly, or approximately), then the matrices give (exactly or approximately) a representation, called embedded.

In physical terms the embedded representation describes the adiabatic coupling; and it explains why some models can be successful, when they (seemingly) have no right to be so.

In nuclear physics the effective $SU(3)$ symmetry proved to be approximately valid in heavy nuclei, where the real $SU(3)$ symmetry is badly broken due to the spin-orbit and other interactions [1]. If a symmetry exists, it has several important physical consequences, *e.g.*, it provides us with selection rules.

Selection rules can be very useful in studying nuclear fragmentations, like, *e.g.*, spontaneous fission, or clusterization in general. In light of the recently discovered neutronless cold fission, these questions are of utmost interest in the structural study of heavy nuclei.

Jarrio *et al.* proposed a procedure to determine the effective $SU(3)$ symmetry of heavy nuclei in the region where the asymptotic Nilsson quantum numbers are valid [1]. Based on their method we have carried out symmetry considerations concerning the relative preference of different exotic binary clusterizations of the ground state of the ^{252}Cf nucleus [3]. The possible clusterizations, however, include not only strongly deformed nuclei, but other types as well. Therefore, some sort of extension of the method of [1] is highly needed for systematic studies.

Here we propose an interpolation method for the determination of the effective $SU(3)$ symmetry between zero and large quadrupole deformations. It is based on the numerical expansion of the Nilsson states of a given deformation in terms of the asymptotic basis, and applying the procedure of [1] for these linear combinations. This method is not based on a rigorous proof, it should be considered as a recipe for the interpolation. Nevertheless, self-consistency arguments show that it gives reliable results both in the case of some light nuclei, where the real $SU(3)$ symmetry is valid, and in the case of heavy nuclei.

The structure of this article is as follows. In sect. 2 we review the determination of the effective $U(3)$ symmetries for large deformation. The prolate case is simply a summary of the method presented in [1], while the oblate case is a new extention of this method based on the same arguments of ref. [1]. Section 3 describes a generalization for small deformations. Then we consider some applications both to light and to heavy nuclei, in order to perform self-

[a] e-mail: hess@nuclear.unam.mx

consistency checks. Finally, some conclusions are drawn in sect. 5.

2 Determination of the effective (λ, μ)'s for large deformations

2.1 Prolate shapes

In ref. [1] a method is given for the determination of the effective $SU(3)$ symmetry based on the occupation of the Nilsson levels of a given deformation. Here we indicate the main steps and use the notation of ref. [1].

If by $\boldsymbol{a}_i^\dagger$ $(i = z, +, -)$ the spherical components of the creation operators of the three-dimensional harmonic oscillator are denoted, the algebra of the $U(3)$ group is given by all possible products $\boldsymbol{a}_i^\dagger \boldsymbol{a}_j$ of the creation and annihilation operators. They are divided into raising, lowering and weight operators. The weights are defined such that an oscillation quantum in "z" has a larger weight than the spherical oscillation quantum "+" and the latter one has a larger weight than "−". The two weight operators of $SU(3)$ are combinations of the number operators. The $SU(3)$ labels (λ, μ) can be obtained by applying the weight operators $(\mathbf{n}_z - \mathbf{n}_+)$ and $(\mathbf{n}_+ - \mathbf{n}_-)$ to the highest-weight state $|\varphi_{\lambda\mu}, \text{h.wt.}\rangle$, where $\mathbf{n}_i = \boldsymbol{a}_i^\dagger \boldsymbol{a}_i$ is the number operator of the "i" degree of freedom. This state satisfies the condition

$$\begin{aligned} \boldsymbol{a}_z^\dagger \boldsymbol{a}_+ |\varphi_{\lambda\mu}, \text{h.wt.}\rangle = \boldsymbol{a}_z^\dagger \boldsymbol{a}_- |\varphi_{\lambda\mu}, \text{h.wt.}\rangle = \\ \boldsymbol{a}_+^\dagger \boldsymbol{a}_- |\varphi_{\lambda\mu}, \text{h.wt.}\rangle = 0, \\ (\mathbf{n}_z - \mathbf{n}_+)|\varphi_{\lambda\mu}, \text{h.wt.}\rangle = \lambda |\varphi_{\lambda\mu}, \text{h.wt.}\rangle, \\ (\mathbf{n}_+ - \mathbf{n}_-)|\varphi_{\lambda\mu}, \text{h.wt.}\rangle = \mu |\varphi_{\lambda\mu}, \text{h.wt.}\rangle. \end{aligned} \tag{1}$$

In general a many-particle Nilsson state, obtained by filling the Nilsson orbitals from below, does not satisfy the requirement of eq. (1). However, in the asymptotic limit some of these conditions are satisfied by a many-particle Nilsson state $|\varphi_0\rangle$:

$$\begin{aligned} \boldsymbol{a}_z^\dagger \boldsymbol{a}_+ |\varphi_0\rangle = \boldsymbol{a}_z^\dagger \boldsymbol{a}_- |\varphi_0\rangle = 0, \\ \mathbf{Q}_0 = Q_0 |\varphi_0\rangle, \end{aligned} \tag{2}$$

where

$$\mathbf{Q}_0 = 2\mathbf{n}_z - \mathbf{n}_+ - \mathbf{n}_- + 3. \tag{3}$$

Here $(\mathbf{Q}_0 - 3)$ is the $(M = 0)$ component of the $SU(3)$ quadrupole operator [1]. The authors of ref. [1] prove explicitly, based on the order of filling of the asymptotic single-particle states, that this condition is fulfilled for large $(\epsilon \geq 0.3)$ deformation.

Using this property and restricting to prolate nuclei, two formulas are obtained, one for the determination of an average value of $(2\lambda+\mu)$ and another one for $\mu(\mu+2)$. The numbers obtained are called *effective* quantum numbers. These are

$$\begin{aligned} \langle 2\lambda + \mu \rangle = \sum_f \langle f | 2\mathbf{n}_z - \mathbf{n}_+ - \mathbf{n}_- | f \rangle, \\ \langle \mu \rangle (\langle \mu \rangle + 2) = 4 \sum_{fe} |\langle e | \boldsymbol{a}_+^\dagger \boldsymbol{a}_- | f \rangle|^2, \end{aligned} \tag{4}$$

where the indices f and e refer to occupied (filled) and empty asymptotic Nilsson orbitals, respectively. The asymptotic Nilsson orbitals are denoted by $|N n_z \Lambda \Sigma\rangle$, where N is the total number of oscillation quanta, n_z is the number of oscillation quanta in the z-direction, Λ is the projection of the orbital angular momentum on the z-axis and Σ is the projection of the spin on the z-axis [4, 5]. The non-zero matrix elements of the second formula in eq. (4) are given by [1]

$$\begin{aligned} \left| \langle N n_z \Lambda + 2\Sigma | \boldsymbol{a}_+^\dagger \boldsymbol{a}_- | N n_z \Lambda \Sigma \rangle \right|^2 = \\ \frac{1}{4}(N - n_z - \Lambda)(N - n_z + \Lambda + 2). \end{aligned} \tag{5}$$

When evaluating (5) one has to pay attention to which orbital is filled and if the final orbital ($|e\rangle$) is empty or not. Λ and Σ can have negative values and each orbital is doubly occupied for the case of even-even nuclei. In the above equations only those oscillator shells enter which are open, *i.e.* closed oscillator shells do not contribute because the net sum of those in eqs. (4) and (5) is zero.

2.2 Oblate shapes

For certain applications we need to extend this method to oblate nuclei as well. For these nuclei a similar procedure can be followed.

Moving from the prolate side to the oblate side of deformation the order of the Nilsson orbitals is inverted. *E.g.*, for the prolate case states with the largest number of quanta along the symmetry axes are lowest in energy, for the oblate case they are the highest. Then, to obtain the effective $SU(3)$ irrep, one does not start from the highest-weight state of $SU(3)$, but from the lowest-weight state, defined by

$$\begin{aligned} \boldsymbol{a}_+^\dagger \boldsymbol{a}_z |\varphi_{\lambda\mu}, \text{l.wt.}\rangle = \boldsymbol{a}_-^\dagger \boldsymbol{a}_z |\varphi_{\lambda\mu}, \text{l.wt.}\rangle = \\ \boldsymbol{a}_-^\dagger \boldsymbol{a}_+ |\varphi_{\lambda\mu}, \text{l.wt.}\rangle = 0, \\ (\mathbf{n}_z - \mathbf{n}_+)|\varphi_{\lambda\mu}, \text{l.wt.}\rangle = \mu |\varphi_{\lambda\mu}, \text{l.wt.}\rangle, \\ (\mathbf{n}_+ - \mathbf{n}_-)|\varphi_{\lambda\mu}, \text{l.wt.}\rangle = \lambda |\varphi_{\lambda\mu}, \text{l.wt.}\rangle, \end{aligned} \tag{6}$$

where "l.wt." stands for "lowest weight". The procedure to follow is completely analogous to the one for prolate shapes, as given in ref. [1]. The results are similar, except that the quantum numbers λ and μ are *interchanged*. Instead of applying raising operators to a many-particle Nilsson state $|\varphi_0\rangle$, in the asymptotic limit, one has to apply lowering operators. The many-particle Nilsson state then satisfies

$$\boldsymbol{a}_-^\dagger \boldsymbol{a}_z |\varphi_0\rangle = \boldsymbol{a}_+^\dagger \boldsymbol{a}_z |\varphi_0\rangle = 0\,, \tag{7}$$

which can be proven using figures of the filling order of the asymptotic states as given in ref. [1]. The equivalent formulas to eq. (4) are now

$$\begin{aligned} \langle 2\mu + \lambda \rangle = \sum_f \langle f | 2\mathbf{n}_z - \mathbf{n}_+ - \mathbf{n}_- | f \rangle, \\ \langle \lambda \rangle (\langle \lambda \rangle + 2) = 4 \sum_{fe} |\langle e | \boldsymbol{a}_+^\dagger \boldsymbol{a}_- | f \rangle|^2. \end{aligned} \tag{8}$$

One has to keep in mind that only the valence particles (which may include several open shells) contribute to the *effective* (λ, μ). In order to describe the transition rates one has to introduce an effective electric charge, as in the $SU(3)$ model for light nuclei [6]. The same has to be done if we are interested in the deformation of the mass distribution. As shown in ref. [7], a good estimate for the effective charge is a value of 2.

3 Extension of the procedure to small deformation

The asymptotic limit of the Nilsson model is reached when the orbitals approach straight lines. This can be seen by inspecting the Hamiltonian of the Nilsson model (for axial symmetric nuclei) given by

$$H = \hbar\omega \mathbf{N} - \hbar\omega r^2 \beta Y_{20}(\theta, \varphi) + C(\vec{\boldsymbol{l}} \cdot \vec{\boldsymbol{s}}) + D\vec{\boldsymbol{l}}^2, \qquad (9)$$

where the strength of the spin-orbit and $\vec{\boldsymbol{l}}^2$ interaction can be obtained from ref. [8].

In eq. (9) the dependence on the deformation β is linear. The factor of β is proportional to the z-component of the quadrupole operator. Diagonalizing the Hamiltonian will give the single-particle orbitals as a function of β. The asymptotic limit is reached when the slope of the single-particle orbitals do not change any more, *i.e.* the individual single-particle orbitals have constant quadrupole moment. (Of course, the nucleus itself increases its total quadrupole moment because single-particle orbitals with larger quadrupole moments are lowered with increasing β and get occupied first.)

By inspecting fig. 1, we observe that for $|\beta| > 0.3$ most of the orbitals are approximately straight lines, *i.e.* the asymptotic limit has been nearly reached. However, the lower the absolute value of the projection $\Omega = \Lambda + \Sigma$ of an orbital is, the less the condition of an asymptotic limit is realized. *This means that the orbitals are not pure, they are mixed and the relevance of the other components beyond the asymptotic component is larger.* Though *in most cases* the dominant part is still given by an asymptotic Nilsson level, the procedure proposed in ref. [1] can only be considered as an approximation.

The intention of what follows is to find an *interpolation* between the range of $|\beta| > 0.3$ to $\beta = 0$. At $\beta = 0$ the resulting effective $SU(3)$ irrep should be $(0, 0)$ and the (λ, μ) values should increase to those obtained for $|\beta| > 0.3$. One possibility, the one we follow, is to use a modified procedure of ref. [1]. We expand each Nilsson orbital at a given deformation β in terms of the asymptotic Nilsson states, *i.e.* when we denote by $|\psi_\alpha\rangle$ a given orbital at deformation β, then

$$|\psi_{N\alpha}\rangle = \sum_{n_z, \Lambda, \Omega} C^{\alpha}_{n_z, \Lambda, \Omega} |N, n_z, \Lambda, \Omega\rangle. \qquad (10)$$

In order to get the effective (λ, μ), in eqs. (4) and (8) for the filled and empty states we use now the expansion (10) in terms of the asymptotic Nilsson states. This

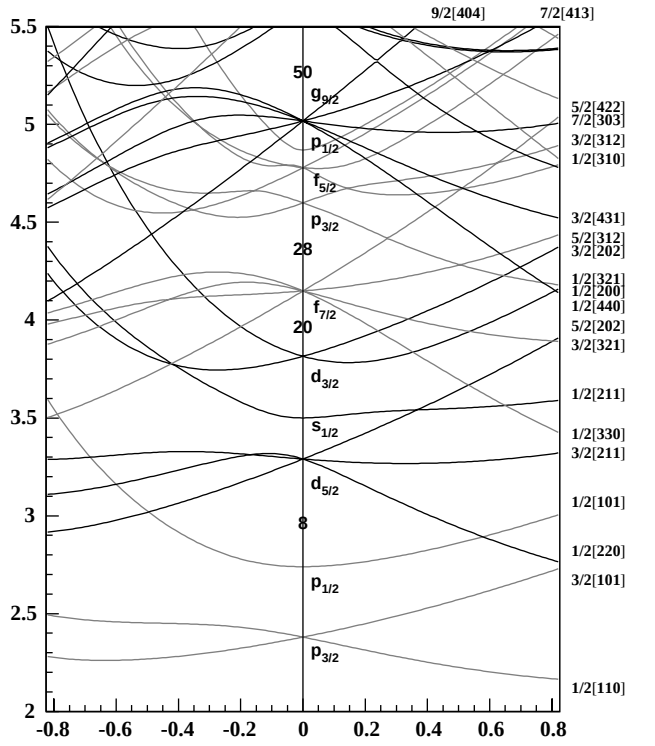

Fig. 1. Nilsson diagram for protons (neutrons), Z or $N < 50$ in the range of $-0.8 \leq \beta \leq 0.8$. Orbitals are denoted by the quantum numbers $\Omega[Nn_z\Lambda]$. The strength of the spin-orbit and $\vec{\boldsymbol{l}}^2$ interaction are taken from ref. [8]. Note that most of the orbitals are nearly straight lines for $|\beta| > 0.3$.

is done by using a numerical routine which is based on an earlier work [9]. *The procedure violates a condition made in* ref. [1] *and should be looked at as a "recipe" for interpolating between large β to zero deformation only.* (Here we refer to the observation of ref. [1] on the filling of the Nilsson orbitals: the states satisfy the condition (2) for prolate and (7) for oblate nuclei. Due to crossing of levels at low deformation this is true only approximately.)

We still have to verify if the final result gives the interpolation searched for. Before doing that we relate the effective (λ, μ) to the deformation of the system. This relation is given in ref. [10], and it is

$$\beta^2 = \frac{16\pi}{5N_0^2}\left(\lambda^2 + \mu^2 + \lambda\mu\right), \qquad (11)$$

(in ref. [10] the square over N_0 is missing), where $N_0 \approx 0.9A^{\frac{4}{3}}$. An analogous expression is also given in ref. [7]. Equation (11) contains already the effective mass charge of 2 [7].

4 Applications

In what follows we assume an arbitrary but given β_{in} (β-initial), fill in the Nilsson orbitals from below at that deformation value and determine the effective (λ, μ). Then

Table 1. Self-consistency check for light nuclei. Starting from the real $U(3)$ symmetry quantum numbers we can determine β according to (11). Then using this deformation we can follow the procedures explained in the text to obtain the new $U(3)$ symmetry quantum numbers. Self-consistency is achieved if the starting quantum numbers (real $U(3)$) are similar to the results obtained.

Nucleus	Real $U(3)$	$\|\beta\|$	Eff. $U(3)_{\text{mix.}}$	Eff. $U(3)_{\text{asym.}}$
^{4}He	[0,0,0]	0.0	[0,0,0]	[0,0,0]
^{12}C	[4,4,0]	0.48	[4,4,0]	[4,4,0]
^{14}C	[4,4,2]	0.20	[4,3,3]	[4,4,2]
^{16}O	[4,4,4]	0.00	[4,4,4]	[4,4,4]
^{20}Ne	[12,4,4]	0.49	[12,4,4]	[12,4,4]
^{24}Mg	[16,8,4]	0.51	[16,7,5]	[16,7,5]
^{36}Ar	[20,20,12]	0.22	[20,19,13]	[20,20,12]
^{40}Ca	[20,20,20]	0.00	[20,20,20]	[20,20,20]

Table 2. Comparison of the results obtained after applying the different approaches (mixing and asymptotic) with real $U(3)$ symmetry quantum numbers. In this case the starting deformations are taken from ref. [11], corrected according to (12). For the ^{4}He a $|\beta| = 0.0$ was used. The effective symmetry quantum numbers are determined according to the relations given in the text.

Nucleus	$\|\beta\|$	Real $U(3)$	Eff. $U(3)_{\text{mix.}}$	Eff. $U(3)_{\text{asym.}}$
^{4}He		[0,0,0]	[0,0,0]	[0,0,0]
^{12}C	0.50	[4,4,0]	[4,4,0]	[4,4,0]
^{14}C	0.32	[4,4,2]	[4,4,2]	[4,4,2]
^{16}O	0.32	[4,4,4]	[4,4,4]	[4,4,4]
^{20}Ne	0.60	[12,4,4]	[12,4,4]	[12,4,4]
^{24}Mg	0.51	[16,8,4]	[16,7,5]	[16,7,5]
^{36}Ar	0.25	[20,20,12]	[20,19,13]	[20,20,12]
^{40}Ca	0.12	[20,20,20]	[20,20,20]	[20,20,20]

eq. (11) will be used to get (in general another) β_{fin} (β-final) deformation. Consistency is achieved if β_{fin} is the same or at least near the β_{in} from which we started. We call *asymptotic approach*, the one introduced by Jarrio *et al.* [1], and *mixing approach*, the interpolation procedure introduced here.

The effective (λ, μ) sometimes contains, *e.g.*, one even and one odd value where both should be even as suggested by the spin content at low energy. For those cases, one has to take the nearest even value instead of the odd value. This problem persists also in the asymptotic limit and has its origin in the fact that the values are only estimations.

4.1 Light nuclei

A simple way for testing the reliability of the new procedure is offered by its application to light nuclei, which have approximately good $U(3)$ symmetry. One can start with the leading $U(3)$ irrep of the ground state of a nucleus, determine its quadrupole deformations by eq. (11), carry out the procedure as described above (or in ref. [1]), and compare the resulting $U(3)$ irreps with the original ones. Obviously, the self-consistency requirement is fulfilled (exactly or approximately) if the results are identical or similar to the starting irreps. We have carried out such calculations for some light nuclei, and the results are shown in table 1.

A further comparison can be done by using deformation parameters from the compilation [11], instead of deriving them from the $U(3)$ irreps. This comparison is shown in table 2.

A fairly good agreement is obtained in both cases.

4.2 Heavy nuclei

As examples for the application to heavy nuclei we consider ^{168}Er and ^{252}Cf. The first one is a well-studied nucleus [1] and the second one attracted a lot of interest recently, due to its decay via cold fission [12]. A first estimate for the experimental deformation β_{in} can be deduced from the tables of ref. [11] (the values of this reference are denoted here by β_{R}) or in case there is no experimental deformation one can take theoretical values from the widely used compilation [13] by Möller *et al.* The β_{R} as given in ref. [11] has to be corrected by

$$\beta_{\text{R}} = \beta_{\text{in}}(1 + 0.36\beta_{\text{in}}) \tag{12}$$

necessary for large deformation [5]. (The reason for this correction is that in ref. [11] the deformation β_{in} is deduced from the $B(E2; 0_1^+ \to 2_1^+)$ value, using eq. (152) of ref. [5] for the Rotation-Vibration Model of the nucleus. This includes a correction factor $(1 + 0.36\beta_{\text{in}})$, not taken into account in ref. [11]. For large deformations, this correction factor gives an important contribution.)

The values of β_{in}, in the ground state of the nucleus, obtained in this way are 0.30 and 0.28 for ^{168}Er and ^{252}Cf, respectively, if we use the compilation of Raman [11], and 0.32 and 0.24, respectively, if we use the compilation of Möller *et al.* [13] which do not require correction.

In order to obtain an overall picture of the approximation, instead of using only the value of β_{in} for ground state, we vary the deformation *artificially* from zero to large values. At each point the effective $SU(3)$ irrep is determined and from the (λ, μ) values the deformation value β_{fin}, as obtained via eq. (11), is deduced. In figs. 2 and 3 (upper panels) we plot the effective $(2\lambda + \mu)$ values obtained for ^{168}Er and ^{252}Cf, respectively, *versus* the deformation value β_{in}. In figs. 2 and 3 (lower panels) we plot the β_{fin} as obtained via eq. (11) *versus* β_{in}. In each case, the upper line gives the result following the procedure as given in ref. [1] while the lower curves show the result of the *recipe* which allows mixing of asymptotic Nilsson levels. The dotted curve in fig. 2 and 3 is given by $\beta_{\text{fin}} = \beta_{\text{in}}$.

As a general feature there is a difference between the two curves even for large β_{in}, indicating that the asymptotic limit has not been reached yet, though the Nilsson orbitals appear to be nearly straight lines. This finding is in agreement with the one of ref. [14]. Furthermore, the

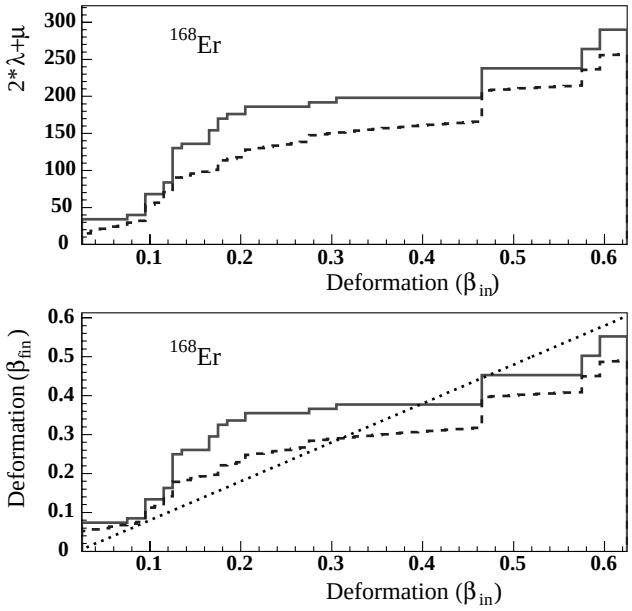

Fig. 2. Self-consistency check of the different approaches for ^{168}Er. The continuous line represents the results for the asymptotic limit (Jarrio's prescription applied to asymptotic states), the dashed line represents the results from our (mixing) method. The upper part of the figure shows the effective $(2\lambda + \mu)$ values as a function of the initial deformation, and the lower part gives the corresponding deformations. See text for more details. In the lower figure, the $\beta_{\rm fin} = \beta_{\rm in}$ line, which corresponds to self-consistency, is represented (dotted line).

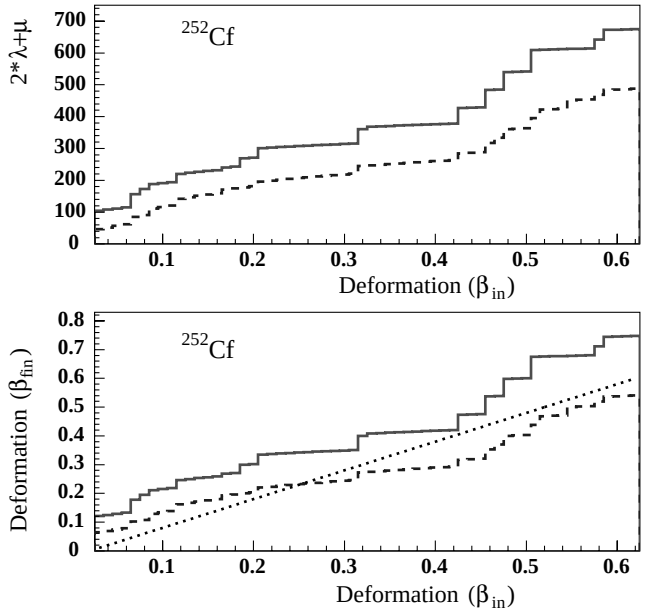

Fig. 3. The same as in fig. 2, now for the nucleus ^{252}Cf.

recipe fulfills better the requirement of self-consistency, *i.e.* it results in a deformation parameter closer to the original one, though the values of the procedure of Jarrio *et al.* [1] are not very far either. Only the "mixing" approach is able to cross the $\beta_{\rm fin} = \beta_{\rm in}$ self-consistency curve for the studied nuclei at approximately the ground-state deformations. At $\beta_{\rm in} = 0$, however, the effective (λ, μ), and therefore β, is not entirely zero but small.

5 Summary and conclusions

In this paper we have proposed a recipe for the extension of the Jarrio *et al.* [1] procedure for the determination of the effective $SU(3)$ symmetry. Our recipe includes two steps: i) a numerical expansion of the Nilsson state in terms of asymptotic basis, and ii) the application of the formulas of ref. [1] for the calculation of the effective $SU(3)$ labels.

This procedure is not based on a rigorous proof, it is only a method for the interpolation between the zero deformation, and the asymptotic region of deformation, where the results are well established. Nevertheless, self-consistency checks support the reliability of our recipe both for light and for heavy nuclei.

The extended method may prove to be a useful tool in the cluster studies of heavy nuclei [15], where it is essential to incorporate the effects of deformation as well as that of the Pauli exclusion principle. This is subject to further investigations by the authors.

Another potential application is given by shell model calculations based on an $SU(3)$ basis [16], especially good for deformed nuclei. In ref. [16] the spectrum of heavy nuclei is obtained, restricting to the pseudo-$SU(3)$ model space [17], treating the nucleons in the unique parity states as spectators. There are, however, several important dynamical properties which have to include explicitly the nucleons in the unique parity states, *e.g.*, backbending. Our contribution can be used to obtain the estimated $SU(3)$ irrep, as done in this contribution, and to use the programs developed by the Louisiana group [18] to get the complete $SU(3)$ content, which in general is quite large for heavy nuclei. Then a cut in the $SU(3)$ basis could be performed around the value of the effective $SU(3)$. We hope that this contribution gives decisive help to these projects.

This work was supported by the CONACyT-MTA and CSIC-MTA exchange programs, and by the OTKA (Grant No. T37502). A.A. acknowledges partial support of the EC HPMFCT-1999-00394 contract.

References

1. M. Jarrio, J.L. Wood, D.J. Rowe, Nucl. Phys. A **528**, 409 (1991).
2. D.J. Rowe, P. Rochford, J. Repka, J. Math. Phys. **29**, 572 (1988).
3. A. Algora, J. Cseh, P.O. Hess, J. Phys. G **24**, 2111 (1998).
4. P. Ring, P. Schuck, *The Nuclear Many Body Problem* (Springer, Berlin, Heidelberg, 1980).
5. J.M. Eisenberg, W. Greiner, *Nuclear Theory I: Nuclear Models*, 3rd edition (North-Holland, Amsterdam, 1987).
6. J.P. Elliott, Proc. R. Soc. London, A **245**, 128; 562 (1958).

7. J. Carvalho, D.J. Rowe, Nucl. Phys. A **548**, 1 (1992).
8. T. Bengtsson, I. Ragnarsson, Nucl. Phys. A **436**, 14 (1985).
9. V. Velázquez, Master Thesis, UNAM, Mexico, 1996, unpublished.
10. D.J. Rowe, Rep. Prog. Phys. **48**, 1419 (1985).
11. S. Raman, C.H. Malarkey, W.T. Milner, C.W. Nestor, D.H. Stelson, At. Data Nucl. Data Tables **36**, 1 (1987).
12. A.V. Hamilton, J.K. Hwang, J.H. Hamilton, A. Sandulescu, A. Florescu, G.M. Ter-akopian, A.V. Daniel, Yu.Ts. Oganessian, G.S. Popeko, W. Greiner, J.D. Cole and GANDS95 Collaboration, Phys. Rev. Lett. **81**, 947 (1998); A.V. Rammaya, J.H. Hamilton, J.K. Hwang and GANDS95 Collaboration, Rev. Mex. Fís. **45** (Supl. 2), 120 (1999).
13. P. Möller, J.R. Nix, W.D. Myers, W.J. Swiatecki, At. Data Nucl. Data Tables **59**, 185 (1995).
14. V. Velazquez, P.O. Hess, J.G. Hirsch, A.E. Mariano, Phys. Rev. C **55**, 1571 (1997).
15. J. Cseh, Phys. Lett. B **281**, 173 (1992); J. Cseh, G. Lévai, Ann. Phys. (N.Y.) **230**, 165 (1994).
16. C.E. Vargas, J.G. Hirsch, J.P. Draayer, Nucl. Phys. A **690**, 409 (2001); **697**, 655 (2002).
17. A. Arima, M. Harvey, K. Shimizu, Phys. Lett. B **30**, 517 (1969); K.T. Hecht, A. Adler, Nucl. Phys. A **137**, 129 (1969).
18. J.P. Draayer, Louisiana State University, $SU(3)$-package, computer code.